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ABSTRACT

The question of the advisability of incorporating discrete mathematics into the mathematics curriculum is addressed by examining the different types of courses typically offered in the first two years of college and the appropriateness of including discrete mathematics topics in these courses. The introductory section explains how the advent of the computer, with its capacity for easily performing extended numerical computations, has made discrete calculations less laborious than continuous functions. This section goes on to enumerate various reasons for teaching discrete math topics. The next sections provide analyses of the advantages and disadvantages of including discrete topics in particular courses, offer examples of ways to present these topics, and review trends in mathematics education with respect to: (1) the calculus sequence; (2) applied calculus; (3) differential equations and linear algebra; (4) finite mathematics; (5) statistics; (6) survey of mathematics for liberal arts students; (7) remedial mathematics; and (8) geometry. A discussion is then presented of the problems associated with incorporating discrete mathematics in the curriculum, such as the national trend towards separate courses on discrete math for math majors; the conservatism of of mathematicians; and the reluctance of the publishing industry to put out innovative texts. A list of discrete math topics is appended. (LAL)

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DISCRETE TOPICS IN THE UNDERGRADUATE MATHEMATICS CURRICULUM: HOW BIG A STEP SHOULD WE TAKE?

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Presented at the Sloan Foundation Conference on New Directions in Two-Year College Mathematics, July 11-14, 1984, Atherton, CA

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DISCRETE TOPICS IN THE UNDERGRADUATE MATHEMATICS CURRICULUM: HOW BIG A STEP SHOULD WE TAKE?

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Introduction.

More than three centuries ago, Newton and Leibniz blazed a new trail in mathematics that led to exciting new vistas on the physical universe. This approach was so immediately successful that every succeeding generation of mathematicians has unquestioningly followed the same path which has led smoothly and continuously to ever new heights and achievements. In the process, the trail has broadened until now it is a twelve lane superhighway covering applications not just in the physical sciences, but also in the biological sciences, the social sciences, business and the management sciences and computer sciences.

Over the last decade, however, there have been some whisperings that the outside lanes of this highway might not provide adequate support to all people using mathematics. It now seems that the very success of the continuous approach may have caused us to overlook other vistas of potentially equal significance and value. In part, this is quite understandable. Prior to the development of the computer, the great difficulty of performing extended numerical computations made it discouraging to treat problems requiring itera-

tive calculations. Not only is this drawback now gone, but there has been a complete reversal whereby discrete calculation is now considerably less labor that calculations with continuous functions. This new capability has now led to considerable discussion and debate (See [3]) about the advisability of incorporating varying amounts of so-called "discrete mathematics" into the mathematics curriculum at all levels.

The present paper will attempt to address this question by examining the different types of courses typically offered in the first two years of college and seeing how appropriate it might be to incorporate discrete topics into existing courses in the math curriculum. Appendix I contains a list of those topics which currently go under the heading of discrete mathematics. Throughout, we will consider the problems associated with actually trying to implement such endeavors.

First of all, let's consider the possible reasons for including discrete topics in mathematics offerings. Basically, they seem to come down to the following:

1. for their own sake;
2. for computer applications;
3. for their use in applications in other subject areas;
4. for their ability to interest and motivate students.

It is the last three items that should dictate to what extent, if any, such curricular changes make sense. The first is primarily an individual judgment.

To gain some perspective, it is a good idea to consider briefly how non-mathematicians view discrete mathematics.

Physicists will tell you that the universe is inherently discrete (quantized) and that a continuum exists only in a mathematician's imagination. Of course, the physicist then goes blithely ahead and uses continuous mathematical models because they work so well. On the other hand, Greenspan [2] has been able to derive virtually all important physical concepts and formulas based solely on discrete formulations involving purely arithmetic and algebraic operations. Further, engineers have realized for years that there are situations which are inherently discrete and so require discrete mathematical methods, the so-called discrete time or sampled data systems. Accordingly, they have developed a wide variety of sophisticated discrete mathematical techniques to handle them. Closer to home, statisticians have dealt with both continuous and discrete methods virtually on a par and have developed parallel mathematical structures for both types of situations.

The Calculus Sequence.

Since the original impetus for continuous mathematics began with the development of calculus, it is only fair to use the same starting point for a discussion of the role of discrete mathematics in the undergraduate curriculum. Several years ago, with support from the National Science Foundation, I developed a new approach to calculus which I called a Discrete Approach [1]. The idea is to incorporate many topics from the calculus of finite differences and sums into the traditional calculus sequence. The rationale for this is:

1. The finite calculus contains discrete parallels to most of the important concepts and methods in calculus. However, since the finite calculus is based on the concept of sequence, its development and applications can be handled using only arithmetic and algebraic operations. This makes the ideas more accessible to weaker students.

2. Once a discrete topic has been treated, the corresponding continuous result can be obtained by resorting to the limit as the step (or increment) approaches 0. Thus, each idea is essentially treated twice. This constant review makes for better student comprehension of the concepts.

3. The non-calculus nature of the development allows solid applications such as discrete max-min analysis and growth and decay problems to be treated within the first week of the course. This provides tremendous motivation to students whose interests are in the use of mathematics, not in the mathematics itself.

4. The discrete setting provides the mathematics necessary to utilize a computer as an integral part of the course - after all, any computer application involves discretizing a function.

To illustrate this discrete approach to calculus, I will describe one of the topics that I cover about one week into the course: finding the approximate location of maxima and minima for a function. By this point, the students have seen the concept of the forward difference operator for a function,

$$\Delta f(x) = f(x+h) - f(x),$$

for any step h. It is then quite apparent to them that when the difference is positive, the function increases (or at least experiences a net overall increase across that subinterval) and when the difference is negative, the function decreases. Further, when the difference is nearly zero, the function is close to its maximum or minimum value. Therefore, the students are able to preview one of the most powerful applications of calculus long before any of the essential machinery is developed and even before being introduced to the concepts of limit and derivative. Further, it is easy to develop the notion of taking smaller values for the step to get a better idea of the location of such extrema, again previewing an essetial element of calculus. Moreover, this approach is a very natural one to implement on a computer using a simple search method.

The results of this project were extremely successful both with my own and various colleagues' classes. Student performance on exams was at least as good. Interest and motivation were considerably higher than usual. In fact, the use of the finite calculus made things so much easier to the students that it was possible to move faster than usual through the course. Thus, there was virtually no loss in other topics to make room for these.

There are a number of very important lessons from my experience with this approach and its implementation which I think we should all bear in mind when considering signifi-

cant changes in the math curriculum. First of all, mathematicians tend to be a very conservative breed, especially when it comes to traditional courses. Further, at many schools, the standard courses such as calculus are offered primarily for the benefit of other departments, such as physics and engineering, and to an extent, we have to structure the courses to their specifications. Finally, courses at two year colleges have to be structured with a view to transferability to four year institutions and this imposes an additional restraint against changes. Together, these factors make for an almost overwhelming inertia that keeps things going in a fixed course. The fact that a better method or approach may exist does not necessarily mean that it will be widely adopted and implemented.

Consequently, if we want to introduce any significant changes in mathematics curricula, then we should focus our attention and efforts on non-traditional math offerings. For example, I believe that many schools are on the verge of developing a new calculus track for computer science majors - there is too much dissatisfaction with the traditional calculus sequence in terms of its relevence to the computer science curriculum. The overwhelming success of calculus in addressing and solving problems from so many areas mitigate against dropping calculus altogether as a requirement for computer science students. On the other hand, virtually all the mathematical work that arises in computer science is basically discrete in nature - continuous problems are simply

discretized to allow the computer to handle them. Therefore, it is reasonable to expect that a compromise will involve the development of a new track that merges a certain amount of discrete parallels into the standard calculus so that the computer science students have an appropriate context in which to use calculus. Further, such a course is likely to use a more intuitive approach based on the sequential notion of limits, difference and sums. This would provide an ideal context in which to develop some of the mathematics (primarily numerical methods) necessary to implement the use of the computer to solve calculus-type problems. It would also demonstrate the connection between calculus and computer science. As such, it would be an effective blend of continuous and discrete mathematics.

Applied Calculus.

Similarly, arguments can be made for the inclusion of other topics of a discrete nature in the various applied calculus courses that are offered to business, social and biological science majors. These are areas where traditional calculus-based mathematics is imposed on situations that are inherently discrete and which at best are approximated by continuous models. For example, discrete and continuous predator-prey models in biology produce different results - which one is based on a more reasonable set of assumptions? Marginal analysis in economics is based on the incremental changes associated with a unit increase in sales or produc-

tion. Is it really appropriate to resort to a limit? In fact, in these and other comparable situations, discrete situations are modeled by continuous processes which, in turn, are solved in practice on computers by discretizing the mathematics. At best, this represents circular reasoning; at worst, it leads to errors in the final results.

Would it not be simpler and more logical to introduce some of the comparable discrete models in the courses for these students? After all, topics such as sequences, recursive relations, and solutions of difference equations are considerably simpler than many of the concepts of calculus and are potentially of greater use. For example, growth and decay problems can be treated in terms of the simple difference equation $A_{n+1} = k A_n$ which leads quickly to the solution $A_n = A_o k^n$. This is far simpler and is more meaningful in terms of the usual applications of such problems than the standard approach involving integration and properties of logarithmic and exponential functions. For that matter, how many banks compound interest continuously?

Differential Equations and Linear Algebra.

Just as the calculus of finite differences and sums is an almost exact mirror image of the usual continuous calculus, the theory and concepts of difference equations are parallel to those of differential equations. As such, there is much that could be gained from a course that encompasses both. This is particularly true of those differential equa-

tions courses which include segments of linear algebra, since the combination of difference and differential equations would provide a pair of complementary systems to which the algebra is applied. Further, the opportunity to experience both continuous and discrete models in a single course would be a valuable opportunity for the students. For instance, the classical discrete and continuous predator-prey models could be studied and the results of each compared.

With this approach, the students would thereafter be in a much better position to select between appropriate discrete and continuous models to find the one that better describes a particular application or problem that might arise in other courses or in their careers. As things stand now, there is an almost total bias towards using purely continous models. (A similar bias is created in most math students towards the use of deterministic models over stochastic models.) Finally, the availability of difference equation notions and the underlying discrete structures involving sequences and recursion relations would be a major advantage in introducing numerical procedures into such a course in a coherent and natural manner.

Despite all of these advantages, I do not see such a course evolving in the immediate future. If nothing else, the present versions of elementary differential equations courses are simply too heavy to leave room for additional material, no matter how good the justification. This applies to traditional equation-solving courses, theoretically oriented

courses emphasizing existence and uniqueness results, computational and computer oriented courses, and those having a linear algebra component. Moreover, the factors dicussed in the previous section on the Calculus sequence certainly apply here as well.

Finite Mathematics

Of all the standard courses offered at the undergraduate level, the one that has come closest to containing significant amounts of discrete mathematics is the finite mathematics course. In fact, the primary difference between finite mathematics and the new discrete mathematics courses is that the former emphasizes applications towards business and related fields. On the other hand, the new courses in discrete mathematics are directed primarily towards the mathematical needs of computer science students and so tend to be somewhat more sophisticated mathematically and to have a greater variety of mathematical topics.

The one change that is likely to occur in the finite math course over the next few years involves an increased use of the computer with particular emphasis on software to handle matrix manipulations. Once the students can have all matrix operations performed for them by machine (or conceivably by a future version of the hand-held calculator), it is not very sensible to force them to perform such operations by hand. Thus, I foresee a reduction in the time spent on matrix operations and related methods such as row reduction opera-

tions. The time saved is likely to be devoted to including other topics of a discrete nature. Incidentally, while I do not feel that students in finite math courses should be required to perform matrix manipulations when appropriate software is available, it is still an essential ability for students in linear algebra.

Statistics.

The nature of the subject matter in the standard non-calculus based elementary statistics course already imposes a considerable emphasis on topics from discrete mathematics. In addition to the discrete distributions such as the binomial, hypergeometric and Poisson, many subsidiary discrete topics such as set theory, counting principles along with permutations and combinations, tree diagrams and sample spaces are integral parts of the course. There is really little justification for including additional topics in discrete mathematics in such a course.

However, it is reasonable to expect that many of these discrete topics will increasingly find their way into the precollegiate mathematics curriculum. Therefore, we can expect to see more students coming into elementary statistics courses who have had a previous exposure to some of the topics we normally cover. Consequently, we should anticipate spending less time on such topics in the future. Additional topics of a purely statistical nature will then have to be added. Alternatively, we might well see some of this extra

time devoted to real-life applications of the subject matter as students conduct experiments and surveys and perform the statistical analyses using simplified statistical software packages. Therefore, we should expect a certain degree of evolution in the content and emphases of such a course over the next few years.

Survey of Mathematics for Liberal Arts Students.

Next to finite mathematics, the traditional survey of math course has probably been the course having the highest discrete mathematical content. This will likely continue. Considering the high interest that the general public and hence many liberal arts students have in computers, it seems reasonable to emphasize them and the mathematics underlying them more heavily in such a course than most of us do today. Thus, set theory, logic, induction, recursion relationships, graph theory, combinatorics, Boolean algebra, probability and the like should remain among the core topics for such courses.

Remedial Mathematics.

Rather than focusing on individual courses in the remedial area, I think that some general comments are in order. There are some significant differences in the reasons such courses are offered in college and in secondary schools. In high school, courses such as algebra and even trigonometry are taken by the "good" students because it is the expected

thing to do and they presumably know what their future academic needs will be. By the time people get to college, they take remedial math courses because of specific requirements of a curriculum or the need to remove an academic deficiency. In either event, the math is not taken for "fun", as a rule, but rather to develop very specific skills that are usually required for other courses in mathematics and related (as well as unrelated) fields. We cannot afford to lose sight of this. Very few of the topics covered by the term discrete mathematics really add to the arithmetic, algebraic and trigonometric skills that these students need to master. (The binomial theorem may be one notable exception.) As a result, I see very little potential impact of discrete topics on these courses.

In a related direction, relatively few students taking remedial mathematics ever reach high level math and science courses. Many are content with, or require no more than, an elementary course in statistics, for example, and are therefore well served by an earlier exposure to some discrete topics. Thus, it might make sense to eliminate some of the more sophisticated manipulative methods from algebra and trig courses. However, this would necessitate developing various remedial tracks depending on the eventual goals of the students and this is a luxury few of us can afford.

There is another fact about these courses that should be considered. A very high percentage of the students we see in remedial math courses have already tried the course in high

school and have either failed or had limited success. Either way, they come into the college courses with certain preconceptions of what algebra or trig is all about and it is usually very difficult to teach them a better or easier or more sophisticated way to deal with problems. (A notable example of this occurs if we expect them to use anything other than the slope-intercept form for the equation of a line.) Further, if we do things too differently, we run the risk of confusing rather than educating such students. This danger can be compounded if we attempt to incorporate additional topics that they "know" are not part of the course.

On the other hand, a good case can certainly be made that an infusion of fresh topics can enliven a class and so motivate students. This might be especially true in the case of arithmetic where topics such as simple combinatorics and elements of probability can be used to reinforce arithmetic skills while making the mathematics more interesting. However, I do feel that such efforts should be approached with caution. While they can make some of the non-obvious applications of mathematics more accessible to weak math students, this should not be done at the expense of the necessary skills they must acquire. Also, students who have trouble mastering simple arithmetic concepts and operations may well find the concepts and verbal problems involved overwhelming.

Such topics and motivation are better suited, in some ways, for inclusion in mathematics curricula at the elementary and junior high school level. There, the teacher has

considerably more time and flexibility available to incorporate new material or new approaches. Further, it makes more sense to attempt to motivate and interest students before they lose interest in mathematics, as is too often the case at the junior high school level. Some of the topics in discrete mathematics certainly have that promise.

Of course, the overwhelming difficulty with this idea is that many of the teachers at that level are not well prepared in mathematics and so may not be willing or able to teach such material. In many ways, the question of mathematical preparation of elementary and secondary teachers presents us with our most important challenge over the coming decade and is one which we must all address.

One final point may be worth mentioning here. Within the next ten years, we will very likely be faced with making some very far-reaching decisions on the content of remedial courses and on the needs of the students taking them. The availability of symbolic manipulation software such as muMath and MACSYMA will have a drastic impact on our curricula and the philosophies underlying them. It is not unreasonable to expect, for instance, the development of a hand- held version of such a package that each student will own and use as he or she now uses a calculator!

Geometry.

The one remedial level math course that was not mentioned in the previous section is geometry. This falls into a

different category in that it is not intended to develop skills on the part of the student. Rather, the usual reasons given for teaching geometry are that it helps develop thinking skills and geometric intuition and provides the student with the set of geometric facts needed in subsequent math courses. For all of this, the course has been dying, both at the high school and college level. Those of us who have been teaching calculus during the last few years have certainly seen the effects of this - the students no longer have any geometric feel for mathematics nor do they have any background in mathematical reasoning and the nature of proof.

As a consequence, it seems to me that geometry is the one place in the remedial curriculum that most needs an infusion of new material or a new approach to regenerate interest. There have been some fascinating reports lately of the use of computer graphics, including use of LOGO, to address this from one direction. For example, many of the major results of geometry can be "discovered" using computer graphics. Thus, students can be led to generalized results a study of individual cases. Similarly, the inclusion of some discrete mathematical topics may also spark a major improvement. For example, some of the modern ideas in geometry, including graph theory and networks, would fit in very nicely and motivate student interest.

Implementing Discrete Mathematics in the Curriculum.

To this point, I have addressed how discrete mathemati-

cal topics may be incorporated into existing math courses. The national trend, however, seems to be in the direction of instituting a self-standing course in discrete mathematics primarily intended for computer science majors. I wonder, though, if math majors would be best served by such an approach. The course would stand by itself and the student would rarely see how discrete mathematics relates to the rest of the mathematical curriculum.

In many ways, this would be analogous to the way that numerical analysis is presently treated. Unless a math student majors in numerical methods, he or she does not see how the subject can and should be integrated into most math offerings from calculus through linear algebra and differential equations and possibly beyond. This lacking creates a severe blind spot in terms of using modern mathematical developments. I fear that the same will hold true of discrete formulations if they are segregated from the rest of mathematics.

Unfortunately, from this point of view, it is much simpler to introduce a new course at any institution, particularly an elective, than it is to change an existing course syllabus. Whenever you add a new topic, something has to go to make room for it. Invariably, people will fight to keep the existing topics (often, though not always, with good reason). Further, mathematicians as a group are extremely conservative - for example, mathematicians have the lowest rate of computer usage in their courses of all the science disciplines. This type of attitude will contribute to resistance

to incorporating new topics and emphases in existing courses.

Another problem in implementing curricular changes involves the attitudes of the publishing industry. Very few companies will chance putting out a book that does not have a ready-made audience. Unfortunately, many people and departments use a textbook to define their courses. This certainly is the easiest route in terms of individual preparation; it may also be the safe route in terms of not making an independent statement or judgment about a course. In either event, most math instructors will be unable to implement desired changes without an undue and needlessly repetitious amount of effort. Just picture the number of different people who have been developing computer software independently for their courses over the last decade. However, that effort has its own built in reward in the sense that the person writing the programs is developing an important skill. Developing a set of course notes to incorporate additional topics is probably more in the nature of drudgery. This lack of appropriate classroom materials to introduce discrete topics could provide a tremendous opportunity for a group such as UMAP/COMAP to fill in with an array of appropriate modules until the publishers see well-defined curricula evolve and become sufficiently widely established.

Summary.

Continuous mathematics is responsible for an incredibly broad range of successes in both mathematics itself and the

disciplines where mathematics is applied. Discrete mathematics has the potential for a comparable impact. However, if it is to have such an impact, then it must be incorporated within the existing structure of mathematical education to become available to the widest possible audience. At the same time, we must bear in mind that such changes should not be made simply for the sake of change and that the teachers who follow our lead may not share our vision, motivation and enthusiasm. The lessons the mathematics community hopefully learned from the forced implementation of the New Math should not be quickly forgotten in a rush to implement new and presumably better pedagogical methods.

References

1. Gordon, S. P. A Discrete Approach to Computer Oriented Calculus, MAA Monthly, 1979.

2. Greenspan, ...(to be located)

3. Ralston, A and G. S. Young, The Future of College Mathematics, Springer-Verlag, New York, 1983.

Appendix

Topics in Discrete Mathematics

Set theory

Permutations

Combinations

Binomial theorem and coefficients

Relations

Graph theory

Trees

Mathematical induction

Equivalence relations

Functions

Truth tables

Difference equations and recurrence relations

Partitions

Recursive definition

Propositional logic

Boolean algebra

Probability

Partial orderings

Numeric functions

Summation notation and concepts

Positional notation

Non-decimal bases and base conversion

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